Forty-Eighth Annual

NEAML New England Championship Math Meet

Friday, May 20, 2022

Conducted by

THE NEW ENGLAND ASSOCIATION
OF MATHEMATICS LEAGUES

CONTEST

Round 1. Arithmetic and Number Theory

1. _____

2. _____

3. _____

1. Write as a reduced proper fraction:

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1 + \cfrac{1 + \cfrac{1 + \cfrac{1}{2}}{3}}{4}}{5}}{1 + \cfrac{5}{1 + \cfrac{4}{1 + \cfrac{3}{1 + \cfrac{2}{1}}}}}}$$

2. In Slovinkia, 90% of the people drink tea, 80% drink coffee, 70% drink orange juice, and 60% drink grape juice. No one in Slovinkia drinks all four of these beverages. What percent of the people in Slovinkia drink at least one of the two juices?

3. A girl has a number of books to pack into parcels. If she packs 2, 3, 4, 5, or 6 books per parcel, there is always one book left unpacked. If she packs 7 books per parcel, none are left over. What is the minimum number of books she has to pack?

Round 2. Algebra I

1. _____

2. _____

3. _____

1. A picture of a Vermont covered bridge was 3 inches longer than it was wide. It was mounted on white poster paper, leaving a 2-inch wide border on all sides of the picture. If the area of the sheet of white poster paper is 76 square inches greater than the area of the picture, find the length of the picture in inches.

2. Find all ordered pairs of real numbers (x, y), with $x \geq y$, such that

$$2x^2 + 2y^2 + x + y = 6$$

and

$$4xy + x + y = -2.$$

3. If each of four numbers is added to the average of the other three, the respective sums are 27, 29, 33, and 37. Find the largest of the four numbers.
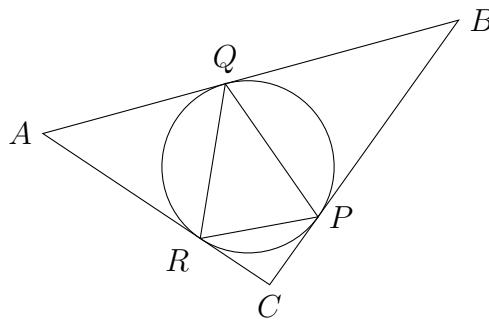
Round 3. Geometry

1. _____

2. _____

3. _____

1. The lengths of the medians of a triangle are 30, 30, and 48. Find the length of the shortest side of this triangle.

2. If triangle ABC circumscribes a circle and the points of tangency are joined to form triangle PQR as shown, what is the measure of $\angle QPR$ if the measure of $\angle A$ is x degrees? Give your answer in terms of x.

3. To cut the stiffest possible rectangular beam of cross-section $GMHL$ from a cylidrical log, a sawyer divides the diameter $\overline{GH}$ at J, O, and K so that $GJ = JO = OK = KH$. He then constructs $\overline{JL}$ and $\overline{KM}$ perpendicular to $\overline{GH}$. He then draws $\overline{GM}$, $\overline{MH}$, $\overline{HL}$, and $\overline{LG}$ and cuts through them. If the diameter $\overline{GH}$ of the cylindrical log is 2 feet long, find the area of rectangle $GMHL$ in square feet.
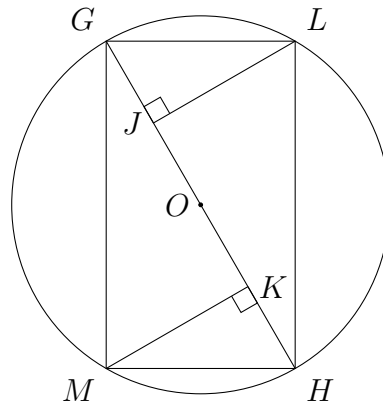
Round 4. Algebra II

1. _____

2. _____

3. _____

1. If $f(x) = ax^5 + bx^3 + cx + 3$, and if $f(9) = -7$, find $f(-9)$.

2. Find x if 3^{3-x}, 3^{-x}, and $3^{\sqrt{9-2x}}$ are in geometric progression.

3. Find the integer k for which

$$\sqrt[3]{1 - 12\sqrt[3]{7} + 6\sqrt[3]{49}} + \sqrt[3]{7} = \sqrt[3]{k}.$$

Round 5. Analytic Geometry

1. _____

2. _____

3. _____

ax^2

1. The graph of a quadratic function $f(x) = a^2 + bx + c$ is a parabola which passes through the points with coordinates $(1, 1)$, $(2, 4)$, and $(5, 1)$. Find the coordinates of the vertex of this parabola.

2. Find all values of m for which the line $y = mx + 5$ will *not* intersect the circle $x^2 + y^2 = 9$ in the real plane. Give your answer in interval notation.

3. A *latus rectum* of an ellipse is a chord of the ellipse which is perpendicular to the major axis and which passes through a focus. If the equation of a conic section is

$$x^2 + 4y^2 - 6x + 16y + 9 = 0,$$

find the area of the rectangle which has its two latus recta as opposite sides.

Round 6. Trigonometry and Complex Numbers

1. _____

2. _____

3. _____

1. Given that a and b are real numbers, $z = a + bi$, and $\overline{z}$ is the complex conjugate of z, find the value of $a^2 + b^2$ if $(z + \overline{z})z = 2 + 4i$.

2. In $\triangle ABC$, $\tan A = 1/2$ and $\tan B = 1/3$. Find the degree measure of angle C.

3. Find the value(s) of θ, $0 \leq \theta \leq \pi$, such that

$$5 \tan \theta - \sqrt{2} \sec \theta + 3 = 0.$$

Give your answer in radians. If necessary, express your answer(s) in terms of an inverse trigonometric function.

Team Round.

1. Find the least positive remainder obtained when 30! is divided by 5^8.

2. Find all real values of x which satisfy
$$\frac{x-1}{x} \leq \frac{x}{x-1}.$$
   Give your answer in interval notation

3. Find the area of a triangle in which the lengths of the three medians are 39, 42, and 45

4. Let $f(x) = ax^2 + bx + c$ denote a quadratic function with discriminant 4. $f(6) = 0$ and $f(0) = 6$. There are two such functions f. Express the coefficients a, b, and c of each function as an ordered triple (a, b, c).

5. A ship is lying somewhere on a line $250\sqrt{2}$ kilometers from and parallel to a (linear) shore. Two LORAN posts established at points P and Q on the shore emit signals which are received by the ship. From the time lapse between the signals it was possible to calculate that the ship is $100\sqrt{6}$ kilometers closer to Q than to P. Taking 1 unit = 50 kilometers, a set of coordinate axes was drawn with the x-axis representing the shore. Wth respect to these axes, the posts were located at $P(2, 0)$ and $Q(10, 0)$. With respect to these same axes, determine the coordinates of the position of the ship.

6. Determine the simplified numerical value of
$$(\sin 10°)(\sin 30°)(\sin 50°)(\sin 70°).$$

ANSWER KEY

Arithmetic and Number Theory

1. $\frac{473}{320}$

2. 100%

3. 301

Algebra I

1. 9 *or* 9 inches

2. $(0, -2)$ and $(3/2, -1/2)$ (either order)

3. 24

Geometry

1. 24

2. $\frac{180-x}{2}$ or equivalent

3. $\sqrt{3}$ *or* $\sqrt{3}$ square feet

Algebra II

1. 13

2. -8

3. 8

Analytic Geometry

1. $(3, 5)$

2. $\left(-\frac{4}{3}, \frac{4}{3}\right)$

3. $8\sqrt{3}$

Trigonometry and Complex Numbers

1. 5

2. $135°$

3. $3\pi/4$

Team Round

1. 312,500

2. $(0, \frac{1}{2}] \cup (1, \infty)$. A correct answer must contain the union symbol or the word "or."

3. 1008

4. $\left(\frac{1}{2}, -4, 6\right)$ and $\left(-\frac{1}{6}, 0, 6\right)$ (either order)

5. $(12, 5\sqrt{2})$

6. 1/16