FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 5 Round 1
Arithmetic: Fractions and Exponents

1.) {78,18,10}

2.) {18,28,20}

3.) {58,59,60}

1.) How many integers x have the property that $x^{\{2,4,6\}}$ is between {1000,100000,10000000} and {5000,500000,50000000}?

2.) Given the least integer value of $n > \{4,5,6\}$ such that the average (arithmetic mean) of $\frac{1}{\{4,5,6\}}$ and $\frac{1}{n}$ is of the form $\frac{1}{b}$ where b is an integer, what is the value of $n + b$?

3.) If a and b are the largest positive two-digit integers possible such that $\frac{32^a}{\{\sqrt[4]{16}, \sqrt[3]{64}, \sqrt[3]{512}\}} = 8^{\frac{4}{3}b}$, find $2a - b$.

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Match 5 Round 2
Algebra 1: Rational Expressions & Equations

1.) {42,38,30}

2.) {2159,2061,1963}

3.) {36,50,45}

1.) The ratio of the sum of $\frac{\{9,7,3\}}{x}$ and $\frac{5}{x-1}$ to the difference between $\frac{4}{x}$ and $\frac{6}{x^2-1}$ can be written in simplest form as $\frac{Ax^2+Bx+C}{Dx^2+Ex+F}$ where $A > 0$ and all coefficients have no common factors larger than 1. Find the value of $|A| + |B| + |C| + |D| + |E| + |F|$.

2.) There are two values of k such that the equation $\frac{5x-17}{x-3} = 18x + k$ has exactly one real solution for x. If the sum of the two values is s and the product is p, find $\{p + s, p + 2s, p + 3s\}$.

3.) Ivan and Sandra both commute to the office. Ivan's commute is 45 miles and Sandra's commute is 36 miles. One day they leave their houses at the same time. Sandra's average speed is {12,20,15} miles per hour less than Ivan's average speed and she arrives at the office {15,18,12} minutes after Ivan. What was Ivan's average speed for his commute to work in miles per hour?

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Match 5 Round 3
Geometry: Circles

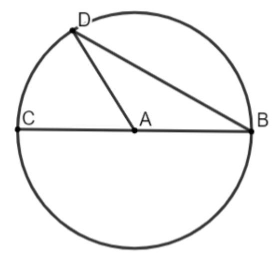
1.) {40,20,30}

2.) {3969,2916,2025}

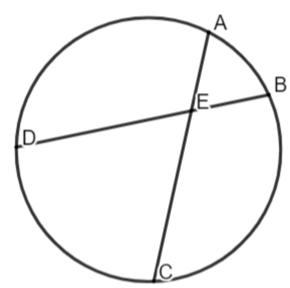
3.) {36,64,100}

NOTE: Figures may not be drawn to scale.

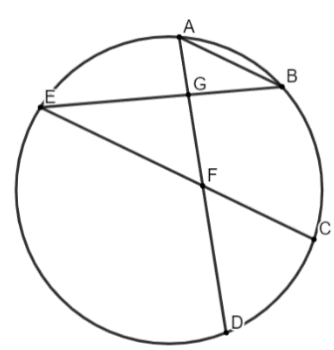
1.) Consider the diagram. The circle has center A and diameter $\overline{BC}$. If the ratio of $m\widehat{BD}$ to $m\widehat{CD}$ is $\{5:4, 7:2, 6:3\}$, find $m\angle DBC$ in degrees.



2.) Consider the diagram where chords $\overline{AC}$ and $\overline{BD}$ intersect at point E. $m\widehat{CD} = 70^{\circ}$, $m\angle AEB = 45^{\circ}$, and the length of $\widehat{AB}$ is $\{7,6,5\}\pi$ units. If the area of the circle is $k\pi$, find the value of k.



3.) Consider the diagram. Chords $\overline{AD}$ and $\overline{BE}$ intersect at point G, chords $\overline{AD}$ and $\overline{EC}$ intersect at point F, and $\overline{AB} || \overline{EC}$. If $AG = \{6,8,10\}$, $GF = \{9,12,15\}$, $BG = x$ and $CF = y$, then there exist constants a and b such that $y = ax^2 - b$ for any value of x. Find the value of $\frac{b}{a}$.



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Match 5 Round 4 Algebra 2: Quadratic Equations and Complex Numbers

1.) $\{69, 49, 45\}$

2.) $\{144, 256, 324\}$

3.) $\{108, 48, 192\}$

1.) If the quadratic equation $ax^2 + bx + c = 0$, where $a, b,$ and c are integers with no common factors greater than 1 and $a > 0$, has one solution of $\left\{3 + \frac{1}{2}i, 2 + \frac{3}{2}i, 1 + \frac{5}{2}i\right\}$, find the value of $2a - b + c$.

2.) The three functions $f(z)$, $g(z)$, and $h(z)$ have the properties that each shares exactly one complex zero with each of the other two, but all three have no complex zeros in common. If $f(z) = z^2 - \{12, 16, 18\}z + \{45, 73, 90\}$, $g(z) = z^2 - \{27 - 36i, 55 - 48i, 72 - 54i\}$, and $h(z)$ is a quadratic function with a leading coefficient of 1, what is the value of the discriminant of $h(z)$?

3.) Let $f(z) = 2z^2 - (12 + 8i)z + 15 + 24i$. For a particular complex number $k = a + bi$ where a and b are real, it is known that $f(k)$ is real and $|k| = \left\{2\sqrt{10}, 2\sqrt{5}, 2\sqrt{17}\right\}$. Find the magnitude of the product of all possible values of a.

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Match 5 Round 5 Precalculus: Trigonometric Equations

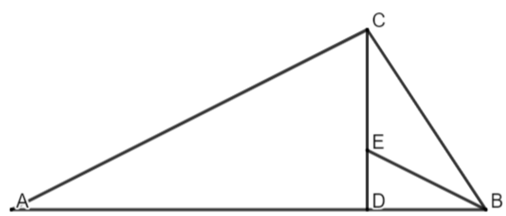
1.) {37,53,62}

2.) {18,32,8}

3.) {1,7,8}

1.) In right triangle FEB with right angle E, if $FE = \{2,3,4\}BE$, find $m\angle FBE - m\angle BFE$ to the nearest whole degree. (Do not round angle measures until the last step.)

2.) Consider the diagram (not drawn to scale). $CD = 1, AD = \{7,9,5\}, BD = \{5,7,3\}$, $\angle CAB \cong \angle CBE$, and $\overline{CD}$ is an altitude of the triangle. If $m\angle EBD = \theta$, find $\cot(\theta)$.



3.) If $\cos\left(x + \frac{\pi}{4}\right) = y$, then there exist constants a and b such that $\sin(2x) = ay^2 + b$ for all x and y. Find the value of $\{2a + 5b, b - 3a, a + 10b\}$

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Match 5 Round 6 Miscellaneous: Sequences and Series

1.) $\{35,53,44\}$

2.) $\{40,32,24\}$

3.) $\{35,39,48\}$

1.) If the first term in an arithmetic sequence is 8 and the sum of the fourth and fifth terms is $\{37,51,44\}$, what is the tenth term of the sequence?

2.) For a particular infinite geometric series, if the sum of the first 1011 terms is half the infinite sum of the series and the difference between the first term and 2023rd term is $\{30,24,18\}$, what is the first term of the series?

3.) A particular arithmetic sequence has a third term of $\{19,20,15\}$ and a fifth term of $\{35,32,23\}$. The sum of the first n terms is S. If $1000 < S < 10000$, how many possible values of n are there?

Team Round

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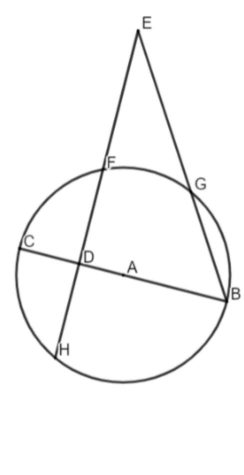
1.) 12	4.) 2700
2.) 30	5.) 6
3.) 18	6.) 130

1.) If $5^{a-b+c} = 10$ and $5^{a+b-c} = 30$, what is the value of 25^{a-1}?

2.) A total cost of $\$N$ is to be split evenly among $P > 4$ people. If 4 people refuse to pay, then each remaining person must pay \$10 more than what they would have paid if everyone contributed. If N is a positive integer less than 10000, how many possible values of N exist?

3.) Consider the diagram (not drawn to scale). The circle has center A and diameter $\overline{BC}$, and $\overline{EH} \perp \overline{BC}$. If the radius of the circle is 13, $AD = 5$, and $GE = 8$, find the value of BG.



4.) For a particular geometric sequence of complex numbers, $a_5 = 3 + i$ and $a_{12} = 25 + 5\sqrt{11}i$. Find the value of $|a_{26}|$.

5.) If $\cos(\theta) + 3\sin(\theta) = 2$, then the sum of all possible values of $\tan(\theta)$ is a and the product of all possible values of $\tan(\theta)$ is b. Find $a - 12b$.

6.) One infinite geometric series has a first term of 7. A second infinite geometric series has a first term of 4. The ratios of the series are opposite numbers, and the infinite sum of the second series is one less than that of the first series. If the first series has a sum that can be written in simplest form as $\frac{a+\sqrt{b}}{c}$ where a, b, and c are integers and b has no perfect square factors greater than 1, find the value of $a + b + c$.