FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 4 Round 1 Arithmetic: Basic Statistics

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1.) Consider the following sequence of numbers: $15, 21, 9, 8, m$, and n. If this sequence has a range of $\{30,28,26\}$ and a median of 14, find the maximum value of the sum of the numbers.

2.) A sequence of consecutive positive integers with a minimum value of a and a maximum value of b has a range equal to its arithmetic mean. If $b \leq \{500,800,1100\}$, find the largest possible value of a.

3.) The geometric mean of a set of n numbers is defined as the nth root of the product of the n numbers. A geometric sequence of 9 numbers has a smallest number of {4,6,9} and a geometric mean of $\{196,150,81\}$. The second largest of the numbers can be written in simplest radical form as $a\sqrt{b}$ with integers a and b. Find $a - b$.

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Match 4 Round 2 Algebra 1: Quadratic Equations

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1.) If a number x is increased by $(4x)\%$, the result is equivalent to $\{2,3,4\}$ more than twice x. This relationship can be represented by the quadratic equation $x^2 + Ax + B = 0$, where A and B are integers. Find $|2A + B|$.

2.) The quadratic equation $x^2 - \{16,14,12\}x + p = 0$ has a solution for x of the form $a + bi$ where $a, b,$ and p are positive real numbers. If $b = a - 1$, find the value of p.

3.) The quadratic equation $\{(a+1)x^2 - 3\sqrt{b}x + 1 = 0, (a-1)x^2 - 3\sqrt{b}x + 1 = 0, (2a+1)x^2 - 3\sqrt{b}x + 1 = 0\}$ has only one real solution for x. If a and b are positive integers less than 100, then the smallest possible value for x can be written as $\frac{\sqrt{m}}{n}$ where m and n are integers and m has no perfect square factors greater than 1. Find $2n - m$.

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Match 4 Round 3
Geometry: Similarity

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1.) A right triangle has a hypotenuse of length 30 and an area of 180. A similar right triangle has a hypotenuse of $\{20,40,50\}$. What is its area?

2.) A conical tank of height $\{24,15,21\}$ feet is filled to the top with water. If one-ninth of the water is drained out, then the height of the water is $a\sqrt[3]{b}$ feet where a and b are integers and b has no perfect cube factors larger than 1. Find $2a - 3b$.

3.) Consider right triangle FCM with right angle C. The altitude from point C intersects the hypotenuse at point L. If $CM = \{2\sqrt{5}, \sqrt{6}, 2\sqrt{3}\}FL$ and the area of triangle FCM is $\{180, 300\sqrt{2}, 150\sqrt{3}\}$, find CL.

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Match 4 Round 4
Algebra 2: Variation

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1.) If y varies inversely as the square of x and $y = 36$ when $x = 8$, find the value of y when $x = \{4,12,16\}$.

2.) The kinetic energy of a moving object varies jointly as its mass and as the square of its speed. If the mass of an object were to increase by $\{60,30,70\}$ percent while its kinetic energy remained constant, by what percent would its speed decrease? Round your answer to the nearest whole percent.

3.) Researchers studying the eastern featherhead bird notice that starting at the age of 2 years, the length of the head feather in centimeters varies directly as the square root of the age in years, and the weight of the bird in ounces varies directly as the cube of the length of the head feather in centimeters. If a four-year-old bird has a weight in ounces that is $\{26,30,34\}$ times the length of the head feather in centimeters, find the ratio of the weight in ounces to the head feather length in centimeters of a 10-year-old bird.

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Match 4 Round 5 Precalculus: Trig Expressions & De Moivre's Theorem

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1.) If A is an angle such that $\sin(A) = \left\{\frac{3}{7}, \frac{5}{9}, \frac{2}{5}\right\}$, then $\cos(2A) = \frac{p}{q}$ where p and q are integers with no common factors greater than 1. Find $p + q$.

2.) One of the complex eighth roots of $\sqrt{3} - i$ lies closest to the {negative y-, positive y-, negative x-}axis compared to the other complex eighth roots. The argument for this number is $\frac{a\pi}{b}$, where a and b are integers with no common factors greater than 1. Find $2b - a$.

3.) If x is an angle in quadrant I such that $\tan\left(x + \frac{\pi}{4}\right) = \tan(x) + \{4,6,8\}$, then $\sec^2(x) = a - b\sqrt{c}$ where a, b, and c are integers and c has no perfect square factors greater than 1. Find $a + 2b + 3c$.

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Match 4 Round 6
Miscellaneous: Conics

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1.) An ellipse with equation $\frac{x^2}{\{100,64,144\}} + \frac{y^2}{b} = 1$ has foci that are 6 units apart. Find the sum of all possible values of b.

2.) Circle P has the equation $x^2 + y^2 - 4x - 6y = 3$. Line l intersects circle P at two points. These points are also shared by Circle Q, which has the same radius as circle P but whose center is shifted {1,2,3} right and {2,3,2} down compared to that of circle P. Line l has equation $Ax + By = C$ where $A > 0$ and A, B, and C are integers share no common factors greater than 1. Find the value of $A - B + C$.

3.) A particular parabola has a focus at (3,5), an x −intercept of $(\{9,7,6\}, 0)$ and a directrix with equation $x = k$. Find the product of all possible values of k.

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2021-2022 Match 4 Team Round

1.)	4.)
2.)	5.)
3.)	6.)

1.) The geometric mean of a set of n numbers is defined as the nth root of the product of the n numbers. A particular sequence of n numbers $\{a_1, \dots, a_n\}$ has the property that $a_m = k^{(-1)^{m+1}m}$ for $m \in \{1, \dots, n\}$ where k is a positive integer. If the sequence has a median of $\frac{730}{162}$ and a geometric mean of p, find the value of $\frac{5}{p^2}$.

2.) The quadratic equation $x^2 + 3x - 5 = 0$. This quadratic has two real zeros. Another quadratic equation $Ax^2 + Bx + C = 0$, where $A, B,$ and C are relatively prime integers, has real zeros that are exactly k times the zeros of the first quadratic where k is an integer greater than 1. If $|A| + |B| + |C| < 1000$, how many values of k are there?

3.) Consider trapezoid $MATH$ with bases $\overline{MA}$ and $\overline{TH}$. $MA = 6, TH = 10$, and $\overline{MA} \cong \overline{AT}$. If diagonal $\overline{HA}$ creates right angle HAT, then HM can be written in simplest radical form as $\frac{a\sqrt{b}}{c}$. Find $a + 2b - c$.

4.) *Best Boxes, Inc.* measures boxes (rectangular prisms) by their "boxiness", which they say varies directly as the square of the length and the square root of their width and inversely as the height. Obviously the boxiness of a box changes with its orientation. A particular box with a volume of 10, three dimensions of different sizes, and one dimension of size 2 has an interesting property: it has the same boxiness when its length is 2 as it does when its width is 2. The length of the box's longest side can be expressed as $\frac{\sqrt[3]{a}}{b}$ where a and b are positive integers and a has no perfect cube factors greater than 1. Find $a - b$.

5.) How many complex numbers z have the property that $z^{100} = 1$ and $z^{2021} = z$?

6.) Two strobe lights that are blinking simultaneously are 275 miles apart. A ship is navigating along a path so that there is a constant difference of $.0005$ seconds between when it receives a signal from either light. If the lights are placed on the x- axis then the path of the ship can be modeled by the equation $\frac{x^2}{A} - \frac{y^2}{B} = 1$. What is the value of B? (Use the fact that the speed of light is $186{,}000$ miles per second. Assume the curvature of the earth is negligible.)