FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 3 Round 1
Arithmetic: Scientific and base notation

1.)

2.)

3.)

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) If the product $2^{\{13,16,10\}} * 49^{2y} * 5^{3x}$ can be written in scientific notation as $1.4 * 10^n$ for some integer value n, find the value of $\frac{x}{y}$.

2.) If n is a positive integer base such that $\{10_n(114_n) = (1000_2)(10_n)(15_n) - 3330_4, 10_n(110_n) = 1001_2(10_n)(14_n) - 10200_4, 10_n(114_n) = 1010_2(10_n)(14_n) - 11010_4\}$, find the sum of all possible values of n. Enter your result as a numeral in base 3.

3.) Consider the repeating decimal $.\overline{aa0}_n$, where n is a positive integer base and a is a digit such that $a = n - 1$. This decimal can be written as a fraction in base 10 as $\frac{p}{q}$ where p and q are numerals in base 10 with no common factors greater than 1. If $p + q = \{181,113,145\}$, find the value of n.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 3 Round 2
Algebra 1: Word Problems

1.)

2.)

3.)

Note: Solutions provided are for Form A only. All forms have similar solution methods.

1.) A boat that would be traveling 20 mph in still water takes {5,8,12} minutes longer to travel against a river's current from Pier A to Pier B than to travel from Pier B to Pier A. If the current has a constant speed of {4,5,10} miles per hour, how far in miles is it from Pier A to Pier B?

2.) Mr. Hill's property contains an equal number of oak trees and maple trees, but the oak leaves take longer to rake. After the maple trees lose half their leaves and the oak trees lose one-third of their leaves, it takes Mr. Hill 50 minutes to rake them all. After all of the remaining maple and oak leaves have fallen, it takes Mr. Hill an additional {80,85,90} minutes to rake them. How many minutes would it have taken Mr. Hill to rake the leaves of all the trees if all of the oak trees were maple trees?

3.) A particular field has grass that grows at a continuous constant rate. If the field starts with {12,30,60} days of grass growth, then it would take 30 cows feeding continuously to clear it in 20 days, or 40 sheep eating continuously to clear it in 36 days. How many days would it take for 10 cows and 20 sheep to clear it with the same amount of starting growth?

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Match 3 Round 3
Geometry: Polygons

1.)

2.)

3.)

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) If the measure of one exterior angle of a regular n-gon in degrees is exactly $\{3.6,2.5,1.6\}n$, find the measure of one interior angle of the n-gon in degrees.

2.) A particular $\{38,65,20\}$-gon has the property that the measures of 5 of its angles in degrees form an arithmetic sequence, with the largest angle having twice the measure of the smallest. The remaining angles are congruent to the smallest angle in the arithmetic sequence. Find the measure in degrees of the largest angle of the polygon.

3.) A particular regular n-gon has interior angles whose difference in measure in degrees from that of one of its exterior angles is k $(k > 3)$ times that of the measure of one of its exterior angles. If the number of diagonals of the n-gon is $\{35,26,15\}$ more than six times the number of diagonals of a k-gon, find the value of n.

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Match 3 Round 4
Algebra 2: Functions and Inverses

1.)

2.)

3.)

Note: The solutions provided are for Form A only. All forms have similar solution methods.

Note: The inverse f^{-1} of a function $f(x)$ is not necessarily a function.

1.) Let $f(x) = \frac{4}{3}x - \left\{\frac{3}{2}, \frac{5}{2}, \frac{1}{2}\right\}$. If (a, b) is an ordered pair on both $f(x)$ and $f^{-1}(x)$, find $2f(2a)$.

2.) The function $f(x) = \frac{1}{\sqrt{x-p}-q}$ has a domain of $[3, \{39,28,19\}) \cup (\{39,28,19\}, \infty)$. What is the value of $q - p$?

3.) Consider the functions $f(x) = \frac{3}{x^2+2x+a}$ and $g(x) = \frac{5}{x^2+ax+b}$. If $f(x)$ has a range of $\left(0, \left\{\frac{3}{2}, \frac{3}{4}, \frac{3}{8}\right\}\right]$ and $g(x)$ has a range of $\left(-\infty, \left\{-\frac{4}{5}, -\frac{4}{9}, -\frac{4}{17}\right\}\right] \cup (0, \infty)$, find the value of $2a - 3b$.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 3 Round 5
Precalculus: Exponents & Logarithms

1.)

2.)

3.)

Note: Solutions provide are for Form A only. All forms have similar solution methods.

1.) If $\log_9(6) = \{3x + 4, 2x + 9, 5x + 1\}$, then $\log_3(8) = ax + b$ where a and b are integers. Find $2a + b$.

2.) For $x > \frac{1}{3}$, the function $f(x) = \log_5(3x - 1) + \log_{25}(2x + \{1,3,5\})$ can be written as $f(x) = A\log_5(ax^3 + bx^2 + cx + d)$. Find the value of $(a + b + c + d)^{1/A}$.

3.) For a particular positive value of n, the equation $\log_x(\{625,1296,256\}) + \log_{\{5,6,4\}}(x^9) = n$ has only one real solution for x. If this solution is $x = k$, find the value of $3k^3$.

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Match 3 Round 6
Miscellaneous: Matrices

1.)

2.)

3.)

Note: Solutions provided are for Form A only. All forms have similar solution methods.

1.) If $\begin{bmatrix} m & 2 & 5 \\ -1 & n & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ m \end{bmatrix} = \begin{bmatrix} \{16,4,-8\} \\ 71 \end{bmatrix}$, find the value of n.

2.) If matrix $A = \begin{bmatrix} 3 & -\frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix}$, matrix $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $AB = A^{-1}$, find the value of $\{a, b, c\}$.

3.) The matrix $A = \begin{bmatrix} \{2,3,7\} & b \\ 5 & a \end{bmatrix}$ has all positive integer entries whose sum is less than 1000 and a determinant of 2. Find the maximum value of the sum of the main diagonal entries of A^{-1}.

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2021-2022 Match 3 Team Round

1.) 4.)

2.) 5.)

3.) 6.)

1.) For particular values x and y, $(6^{2x})(18^{y})(50^{x+y}) = 60^{n} = a * 10^{14}$, where n is a positive integer and $1 \leq a < 10$. Find the value of $x - y$.

2.) Maddy and Patty go for a run. Maddy runs the first half of her total running distance at an average speed of m miles per hour but then slows down to an average speed of $\frac{1}{2}m$ miles per hour for the remainder of the run. Patty spends the first half her total time? running at an average speed of p miles per hour but then slows down to an average speed of $\frac{1}{2}p$ miles per hour for the remainder of the run. If Maddy and Patty end up running the same total distance in the same total time, find the value of $1000\left(\frac{m}{p}\right)$.

3.) A regular m-gon and a regular n-gon, where $m \geq n$, have the property that the sum of the measures of one interior angle of each in degrees is equal to the number of diagonals in a 27-gon. If A is the value of m where $m - n$ is maximized, B is the value of m where $m - n$ is minimized, and C is the number of ordered pairs (m, n) that exist, find the value of $A + B + C$.

4.) The function $f(x) = 2x - 9$ is reflected across $y = \frac{1}{2}x$ to product the function $g(x)$. Find the value of $g(-27)$.

5.) If the function $f(x) = \log_a(2 + 4x - x^2) + \log_a(10 - 4x + x^2)$ has a maximum value of $\frac{1}{3}(\log_2(3) + 1)$, what is the value of a?

6.) The nonsingular matrix $A = \begin{bmatrix} k & 3 & 5 \\ 2 & k+1 & -1 \\ 2 & 0 & 4 \end{bmatrix}$ has the property that the determinant is equal to the trace (sum of the main diagonal entries) of the matrix. What is the value of the determinant?