FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 2 Round 1 Arithmetic: Factors & Multiples

1.) __________________

2.) __________________

3.) __________________

1.) The least common multiple of a and b is $\{6,5,4\}$ times the greatest common factor of a and b. If $ab = \{864,720,576\}$, find the least common multiple of a and b.

2.) What is the smallest whole number to have exactly $\{44,45,50\}$ factors, including 1 and itself?

3.) The greatest common factor of N and $\{54,24,54\}$ is $\{9,4,9\}$. The least common multiple of N and $\{378,120,594\}$ is $\{1890,1320,4158\}$. Find the sum of all possible values of N.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 2 Round 2
Algebra 1: Polynomials
and Factoring

1.) ________________

2.) ________________

3.) ________________

1.) If $(2x+3)(ax+b) = \{8,6,4\}x^2 + cx - \{21,15,27\}$ for all values of x, find the value of $a - 3b - 2c$.

2.) If the polynomial $2x^3 - \{22,26,20\}x^2 + mx - n$ with constant coefficients m and n has three not-necessarily distinct positive integer zeros, what is the largest possible value of n?

3.) A particular quartic polynomial with integer coefficients has a leading coefficient of 1, a cubic coefficient of $\{-12, -16, -14\}$, one zero of $\{2+i, 3+i, 2+2i\}$, and another zero of $a + bi$ where a and b are nonzero integers and $a \neq \{2,3,2\}$. If the constant term of the quartic is less than 2021, find the number of possible values of b.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 2 Round 3
Geometry: Area & Perimeter

1.) ________________

2.) ________________

3.) ________________

1.) A square with area N has a perimeter equal to the circumference of a circle with diameter {6,8,10} and area M. Find the value of $\frac{M^2}{N}$.

2.) A regular hexagon has the property that the difference between the longest diagonal length and the perpendicular distance between any two opposite sides is exactly {7,6,5} units. The perimeter of the hexagon can be written as $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + 2b + 3c$.

3.) Consider trapezoid $MATH$ with $\overline{MA}||\overline{HT}$, right angle H, $MA < TH$, $MA = \{16,18,16\}$, $MH = \{7\sqrt{7}, 12\sqrt{5}, 24\sqrt{3}\}$ and $AT = \{4\sqrt{23}, 6\sqrt{21}, 16\sqrt{7}\}$. Point Y lies on diagonal $\overline{MT}$ such that $\overline{AY} \perp \overline{MT}$. Find $(AY)^2$.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 2 Round 4 Algebra 2: Absolute Value & Inequalities

1.) ____________________

2.) ____________________

3.) ____________________

1.) How many integers satisfy the inequality $3|x - 11| < \{136,133,130\}$?

2.) The inequality $|x + a| < b$ has a solution set for x of $(a - 2, -5a - \{3,1,5\})$. Find the value of b.

3.) Consider three positive numbers a, b, and c such that $a < b < c$. The minimum value for $x \in [a, c]$ of $f(x) = |x - a| + |x - b| + |x - c|$ is $\{17,20,25\}$. The maximum value of $f(x)$ for $x \in [a, c]$ is $\{30,33,40\}$. Find the value of $|a + c - 2b|$.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 2 Round 5
Precalculus: Law of Sines & Cosines

1.) ________________

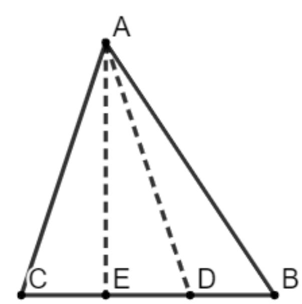
2.) ________________

3.) ________________

1.) Given triangle MRH with $m = 2$, $r = 3$, and $h = 4$, $\sin^2(\{M, R, H\}) = \frac{a}{b}$ where a and b are integers with no common factors greater than 1. Find $b - a$.

2.) Consider triangle ABC with area $\{42\sqrt{5}, 10\sqrt{21}, 6\sqrt{5}\}$. If $\sin(C) = \left\{\frac{3\sqrt{5}}{7}, \frac{\sqrt{21}}{5}, \frac{\sqrt{5}}{3}\right\}$ and both a and b are integers, find the positive difference between the maximum possible value of c^2 and the minimum possible value of c^2.

3.) Consider triangle ABC with acute angle B and distinct points D and E on $\overline{BC}$ such that $BD = DE = EC$. If $AB = \{10,15,20\}$, the area of triangle ABC is $\{36,54,72\}$, and $\sin(B) = \frac{24}{25}$, then $\sin(\angle AEB) = \frac{a\sqrt{b}}{c}$ where a, b, and c are integers with a and b having no common factors greater than 1 and b having no perfect square factors other than 1. Find $a + b + c$.



FAIRFIELD COUNTY MATH LEAGUE 2021-22

Match 2 Round 6 Miscellaneous: Equations of Lines

1.) ___________________

2.) ___________________

3.) ___________________

1.) A line perpendicular to $3x - Ay = 24$ but with the same x-intercept has equation $Bx + Cy = \{40, 32, 56\}$, where A, B, and C are positive numbers. Find the value of AC.

2.) A line with a positive slope can be written parametrically as $x = at + 1$ and $y = 6t + b$. If a and b are integers and the line contains the point $\{(10,3), (8,5), (9,7)\}$, find the sum of the greatest possible values of a and b.

3.) A nonzero number m has the property that if a line has a slope of m, any line perpendicular to it will have a slope exactly $\{3,4,5\}$ less than m. Line a has slope m^2 and y-intercept $(0, -34)$. Line b is perpendicular to a and has y-intercept $(0, \{50,120,127\})$. Find the x-coordinate where lines a and b intersect.

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2021-2022 Match 2 Team Round

1.) ___________________ 4.) ___________________

2.) ___________________ 5.) ___________________

3.) ___________________ 6.) ___________________

1.) The greatest common factor of 12 and N is 4. If there are at least 175 positive integers less than or equal to 2021 that are divisible by 12 or N find the largest possible value of N.

2.) Consider $f(x) = x^4 - 5x^2 + 4$. For how many positive integer values of $n \leq 1000$ is $f(n)$ divisible by 360?

3.) Quadrilateral $FCML$ is inscribed in a circle with an area of 50π, and $\overline{FM}$ is a diameter of the circle. The altitude of triangle FCM from C intersects $\overline{FM}$ at D, and the altitude of triangle FLM from point L intersects $\overline{FM}$ at E. If $ED = 4DM$ and $FE = 5DM$, find the area of $FCML$.

4.) If, for constants a and b, the solution set for $|x - ab| > b$ is $\left(-\infty, -\frac{2}{3}a\right) \cup \left(\frac{3}{2}b, \infty\right)$, find the value of $10a + 15b$.

5.) On a particular day, an 8 foot pole casts a 6 foot shadow on level ground when the pole is inserted perpendicular to the ground. At the same time, an identical 8-foot pole also standing perpendicular to level ground casts a five foot shadow on a hill with an angle of elevation $\theta < 45^{\circ}$ to level ground. If $\sin(\theta) = \frac{a}{b}$ where a and b are integers with no common factors greater than 1, find $a + b$.

6.) The line $x + 3y = 9$ intersects a circle centered at the origin with radius 5 at two points, creating a chord with endpoints D in quadrant I and E in quadrant II. If point F is placed on the circle such that $DE = EF$, then the line containing the points D and F has equation $Ax + By = C$, where $A > 0$ and $A, B,$ and C are integers that share no common factors greater than 1. Find $A + B + C$.