FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 1 Round 1
Arithmetic: Percents

1.)

2.)

3.)

1.) How many natural numbers less than 100 produce a natural number when increased by $\{40\%, 25\%, 33\frac{1}{3}\%\}$ and then again by $\left\{33\frac{1}{3}\%, 16\frac{1}{6}\%, 15\%\right\}$?

2.) One item in a store is discounted by 70% and then by an additional 20%. Another item undergoes a price increase of $\{20,60,50\}\%$ and then a decrease of $x\%$. If the total percent discount on both items ends up being the same, find the value of x.

3.) For positive numbers A, B, and C, it is known that $A\%$ of B is equal to the difference between 30% of A and 40% of B. If $A\%$ of C is $\{13,11,14\}$ and $B\%$ of C is $\{6,7,9\}$, find the value of the product ABC.

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Match 1 Round 2
Algebra 1: Equations

1.)

2.)

3.)

1.) If $x = 4$ is the solution to the equation $\{38,52,47\}x + 192 = kx + 8$ where k is a constant, find the value of k.

2.) If p and q are positive integers such that $11p - \{3,5,7\}q = \{2,1,4\}$, and $q < 1000$, find the largest possible value of p.

3.) If the equation $\sqrt{x^2 + ax} = x + \{14,10,6\}$ has a positive integer solution for x, find the second-largest possible value of a.

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Match 1 Round 3
Geometry: Triangles & Quadrilaterals

1.)

2.)

3.)

1.) A quadrilateral has angles whose measures in degrees form an arithmetic sequence. If the second largest angle measures $\{100,105,110\}$ degrees, what is the measure of the largest angle in degrees?

2.) A rhombus has one diagonal that is three times the length of its other diagonal and an area of $\{540,240,60\}$. What is its perimeter?

3.) An isosceles trapezoid has a height of $\{3\sqrt{2}, 2\sqrt{3}, 2\sqrt{2}\}$ units and a diagonal length of $\{2\sqrt{17}, 2\sqrt{30}, 4\sqrt{5}\}$ units. What is the area of the trapezoid in square units?

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Match 1 Round 4
Algebra 2: Simultaneous Equations

1.)

2.)

3.)

1.) If (p, p) where p is a constant is the solution for (x, y) to the system $\begin{cases} Ax + 2y = \{16,21,4\} \\ 3x - Ay = \{9,19,1\} \end{cases}$ where A is a constant, find the value of p.

2.) Consider the system $\begin{cases} \{2,3,9\}x + my = 2m \\ (2m+1)x + 5y = \{2m+6, 2m+5, 2m+1\} \end{cases}$ where m is a constant. If the system has no solutions for (x, y), find the value of $4m^2$.

3.) Three particular real numbers have a sum of $\{5\sqrt{3}, 6\sqrt{2}, 3\sqrt{15}\}$. The sum of the three products of two of the three numbers is $\{20,15,44\}$. Find the sum of the squares of the numbers.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 1 Round 5
Precalculus: Right Triangle Trigonometry

1.)

2.)

3.)

1.) Consider right triangle TRI with right angle R. If $\sin(T) = TI = \left\{\frac{2}{3}, \frac{3}{4}, \frac{5}{6}\right\}$ then $TR = \frac{x\sqrt{y}}{z}$ where x and z have no common factors greater than 1 and y has no perfect square factors greater than 1. Find $x + y + z$.

2.) A right triangle has the property that the tangent of one of its acute angles is $\{75\%, 80\%, 40\%\}$ larger than its sine. If the area of the triangle is $\{18\sqrt{33}, 20\sqrt{14}, 80\sqrt{6}\}$, find the length of the hypotenuse of the triangle.

3.) In right triangle ABC with right angle C, the sum of $\sin(A)$ and $\cos(A)$ is $\left\{\frac{2\sqrt{3}}{3}, \frac{2\sqrt{10}}{5}, \frac{\sqrt{42}}{6}\right\}$. If the triangle has an area of $\{4,9,5\}$ square units, find the square of the length of the hypotenuse.

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Match 1 Round 6
Miscellaneous: Coordinate Geometry

1.)

2.)

3.)

1.) If a particular line has an x-intercept of $(-1,0)$ and the line passes through $\{(10,66), (8,45), (12,52)\}$, and its equation in standard form is $Ax + By = C$ where A, B, and C are integers with no common factors greater than 1 and $A > 0$, find the value of $A - B - C$.

2.) The circle with equation $(x-3)^2 + (y+k)^2 = r^2$ has a diameter with endpoints $(2,-5)$ and $(a, \{-11,-13,-15\})$. Find the value of $k - a + r^2$.

3.) An isosceles triangle is graphed on the coordinate plane such that its base has coordinates $\{(-3,-3), (-1,-1), (-6,-6)\}$ and $\{(-13,-13), (-9,-9), (-12,-12)\}$. Its vertex has coordinates (p,q). If the triangle has area $\{10\sqrt{7}, 8\sqrt{3}, 6\sqrt{10}\}$, find the value of pq.

FAIRFIELD COUNTY MATH LEAGUE 2021-2022 Match 1 Team Round

1.) 4.)

2.) 5.)

3.) 6.)

1.) How many positive integers n have the property that decreasing n by 25% or increasing n by 20% results in an integer less than or equal to 1000?

2.) If the equation $\frac{ax+2}{x+1} = \frac{cx-\frac{6}{x+1}}{bx-3}$ has infinite solutions for x and both a and b are nonzero, how many different integer pairs (a, b) exist?

3.) Consider parallelogram $FCML$ with midpoint D on $\overline{FC}$. If $m\angle F = 120^{\circ}$, $FL = 10$ and $LD = 14$, then the area of the parallelogram can be written as $a\sqrt{b}$ where b has no perfect square factors larger than 1. Find the value of $a - b$.

4.) Given the system $\begin{cases} Ax + 5y = 60 \\ \frac{5}{x} + \frac{6}{y} = 2 \end{cases}$, the set of all constants A such that the system has no real solutions for (x, y) is bounded below by m and above by n, where m and n are positive integers. Find the value of $2n - 3m$.

5.) In right triangle ABC with larger acute angle A the ratio of $\tan(A)$ to $\sin(A)$ is the square of the ratio of $\sin(A)$ to $\cos(A)$. Find the value of $1000\sec(A)$ rounded to the nearest whole number.

6.) A line with a y-intercept of $(0,16)$ lies tangent to the circle $x^2 + y^2 = 80$ in quadrant 1. The line intersects the circle at the point (a, b). Find the value of b.