FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 4 Round 1 Arithmetic: Basic Statistics

1.) _____

2.) _____

3.) _____

1) What is the positive difference between the arithmetic mean and the median of the set {prime numbers between {55, 65, 75} and 100}? Round to the nearest whole number if necessary.

2) The upper quartile of a set of data is the median of the upper half of the set. The lower quartile of a set of data is the median of the lower half of the set. What is the sum of the upper quartile, the median, and the lower quartile of the set of numbers {multiples of {6,8,4} between 1 and 99}?

3) You have a collection of quarters and dimes. If you add 30 nickels to your collection, you will have {150,180,210 } coins altogether and the mean value per coin will decrease by $\left\{\frac{3}{2},\frac{5}{3},\frac{5}{3}\right\}$cents. How many quarters do you have?

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Match 4 Round 2
Algebra 1: Quadratic Equations

1.) _____

2.)_____

3.) _____

1) If R is the positive root of $\{9,4,2\}(\{2,3,4\}x^2 - x) = \{20,1,3\}$ and T is the negative root of $\{9,4,2\}(\{2,3,4\}x^2 - x) = \{20,1,3\}$, what is {10R+4T, 7R+9T, 8R+10T} ?

2) A quadratic equation has solutions $\{\frac{4+5\sqrt{2}}{3}$ and $\frac{4-5\sqrt{2}}{3}$, $\frac{3+5\sqrt{2}}{3}$ and $\frac{3-5\sqrt{2}}{3}$, $\frac{1+5\sqrt{2}}{3}$ and $\frac{1-5\sqrt{2}}{3}\}$. The equation can be expressed as $Ax^2 + Bx + C = 0$, where *A*, *B*, and *C* are relatively prime integers and $A > 0$. Find $|A + B + C|$.

3) Find the product of all values of k for which the equation $\{(k-8)x^2 + kx + (k-3)$, $(k-15)x^2 + kx + (k-8)$, $(k-21)x^2 + kx + (k-5)\} = 0$ has exactly one real solution for x.

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Match 4 Round 3
Geometry: Similarity
Note: Diagrams are not
Necessarily drawn to scale

1) _____

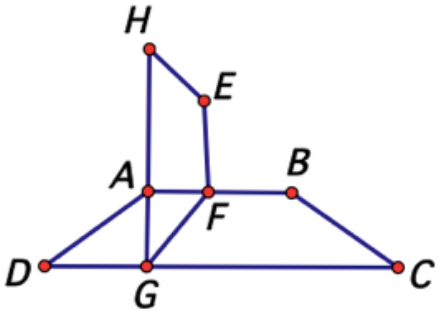
2.) _____

3.) _____

1) The ratio of the areas of two regular octagons is {16:9, 25:9, 49:9}. One side of the smaller octagon measures 6 cm. Find the perimeter of the larger octagon.

2) $\triangle VWX$ is a right triangle with the right angle at X. Point Y lies on segment VW and point Z lies on segment VX. Segment YZ is parallel to segment WX. WX = {5, 11, 7}, $VX = 3\sqrt{3}$, and YZ = 4. As a radical expression in simplest form, $VY = \frac{A\sqrt{B}}{C}$. Find $A + B + C$.

3) In the diagram below, not necessarily drawn to scale, isosceles trapezoid ABCD is similar to trapezoid EFGH with segment AB parallel to segment CD. AB+CD={28,84,56} and 11(AB)=3(CD). HG ^ DC and AB ^ EF. BC={ $4\sqrt{5}, 12\sqrt{5}, 8\sqrt{5}$}. Find the area of EFGH.



FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 4 Round 4
Algebra 2: Variation

1.) _____

2.) _____

3.) _____

1.) $(z-10)$ varies inversely with the square root of w. If z=8 when w=64, what is the value of z when w={256,16,4} ?

2.) m varies inversely with n, and n^2 varies directly with p^3. If p=25 when m=27000, what is p when m={8,27,15625} ?

3.) The electromagnetic force in notwens between two charged particles in Universe X varies jointly with the product of their charges in bmoluocs and inversely with the cube of the distance in retems between them. Particles A and B are 2 retems apart. The electromagnetic force between them is {3600,6000,4800} notwens. Particle B has charge 200 bmoluocs. The force between particles A and C is 120 notwens, and particle C has charge 60 bmoluocs. The distance between particles A and C in retems in simplest radical form is $M\sqrt[3]{N}$. What is M+N?

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 4 Round 5
Trig Expressions and DeMoivre's Theorem

1.) _____

2.) _____

3.) _____

1.) For how many integers N with $1 \leq N \leq \{300, 325, 350\}$ is $\cos N°$ positive?

2.) If $r \operatorname{cis} \theta$ means $r(\cos\theta + i \sin\theta)$, for how many integer values of A between $\{-10$ and $0,\ -5$ and $5,\ 0$ and $10\}$ is $\left(4 \operatorname{cis} \frac{\pi}{A}\right)\left(6 \operatorname{cis} \frac{\pi}{A+3}\right)$ a complex number whose real part is equal to zero?

3.) Lines L_1 and L_2 both have positive slope and pass through the origin. Line L_1 also passes through the point $\{(7, 2), (8,3), (9,4)\}$. The angle between lines L_1 and L_2 is equal to the angle between line L_1 and the positive x-axis. The slope of line L_2 is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 4 Round 6
Conics

1.) _____

2.)_____

3.)_____

1.) A line with a negative slope is tangent to the circle $x^2 + y^2 = 25$ at the point T. The line also passes through {A(8,0), A(9,0), A(7,0)}. Find the square of the length AT.

2.) A hyperbola has foci at $(\{3,4,5\}\sqrt{5}, 0)$ and $(-\{3,4,5\}\sqrt{5}, 0)$ and one of its asymptotes has equation $y = 2x$. The hyperbola intersects the positive *x*-axis at $(a, 0)$. Find a.

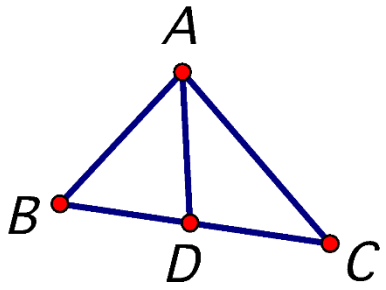
3.) An ellipse has foci at $(2\sqrt{21}, 0)$ and $(-2\sqrt{21}, 0)$. It intersects the *y*-axis at (0, 4) and (0, −4). One intersection point of this ellipse with the circle centered at (0,0) with radius 6 has *y*-coordinate $\{\frac{16\sqrt{A}}{A}, \frac{A\sqrt{21}}{21}, \frac{A^2\sqrt{21}}{21}\}$. Find A.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021 Match 4 Team Round

1) The geometric mean of a set of N numbers $\{a_1, a_2, ..., a_N\}$is defined to be $\sqrt[N]{a_1 a_2 a_3 ... a_N}$. A particular set of six numbers has geometric mean equal to 3. Five of the numbers are $\frac{7}{9}$, $\frac{1}{27}$, $\frac{1}{49}$, 81, and 243. What is the sixth number?

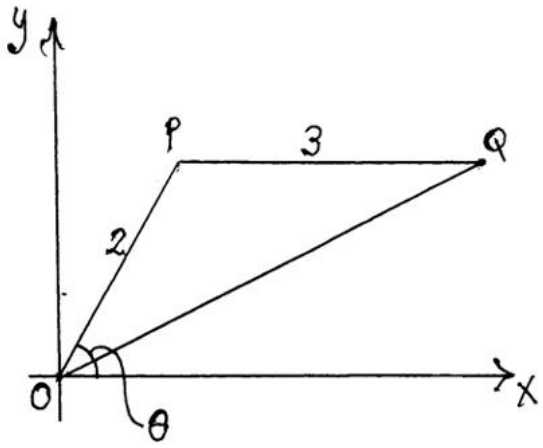
2) For how many integer values of k does the equation $kx^2 + 3x + k = 25$ have two rational solutions?

3.) In the diagram of ΔABC below, not necessarily drawn to scale, line AD bisects $\angle BAC$. AC=6x+3, DC=2x+7, BC=7x-3, AB=4x+2. Find the value of x.



4.) The graph of a direct variation function of the form $y = kx^n$ passes through the points (64,8) and $(4,\frac{1}{128})$. What is $\frac{n}{k}$?

5. In the diagram, line segment OP makes an acute angle θ with the x-axis and has length 2. Segment PQ is parallel to the x-axis and has length 3. The square of the length of segment OQ is $a + b\cos^2\frac{\theta}{2}$. Find $a + b$.



6.) A circle has equation $x^2 - 6x + y^2 + 10y = 2$. A parabola has its vertex at the center of the circle. The focus of the parabola has the same x-coordinate as the x-coordinate of its vertex, and the y-coordinate of the focus is 3 greater than the y-coordinate of the vertex. The y-coordinate of either intersection point of the two curves has form $A + B\sqrt{2}$ for integers A and B. Find $B - A$.