FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Round 1
Geometry: Lines and Angles

Note: Figures not necessarily
Drawn to scale

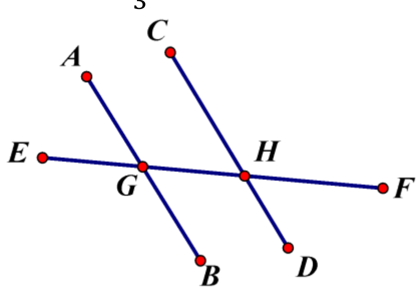
1.) __________{30, 15, 75} degrees___

2.) ________{67, 63, 59}__degrees____

3.) ___________{8, 4, 2}_________

1.) What is the degree measure of the acute angle formed by the intersection of the lines {$y = \sqrt{3}(x-2) + 1$, $y - 4 = x - 5$, $x + y = 3$} and $3y - 3 = \sqrt{3}(x-2)$?

2.) In the figure below, segment AB is parallel to segment CD. The lines are cut by transversal line EF, which intersects line segment AB at G and segment CD at H. There is a number *x* such that the measure of angle AGE is {$(\frac{5}{2}x - 23)$, $(\frac{5}{2}x - 42)$, $(\frac{5}{2}x - 61)$} degrees and the measure of angle GHD is $(\frac{2}{3}x + 89)$degrees. Find the measure of angle FHD.



3.) Rhombus WXYZ has W at {(-4,3), (-8,6), (-6,8) } and X at (0,0). Y is in quadrant I and the slope of line XY is { $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{3}$} . The slope of XZ is {$a + b\sqrt{5}$, $a + b\sqrt{10}$, $a + b\sqrt{10}$ } for some integer values of a and b. Find a+b.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Round 2 Algebra: Literal Equations

1.) ______ {1,3,4}______________

2.) _______{ 14, 6, 36}__________

3.) ______ {1,3,2}_____________

1.) If the equation $\frac{1}{2}(x+3y-8)-z=3z-2(4x-y)$ is solved for z in terms of x and y, then z=ax+by+c. Find {a+b+c, a+b-c, a+b-2c}.

2.)_ If the equation { $x=\frac{y}{2}+\frac{1}{y}$, $x=y+\frac{1}{y}$, $x=\frac{y}{4}+\frac{1}{y}$ is solved for y, $y=\frac{ax\pm\sqrt{bx^2+c}}{2}$ for some values of a, b, and c. Find a + b - c.

3.) Suppose that $x > 0$. When the equation { $2xy(x^2+1)-3x^2=(2x^2+1)(2x^2-1)-x^2y-y$, $4xy(x^2+1)-15x^2=(4x^2+1)(4x^2-1)-x^2y-y$, $3xy(x^2+1)-8x^2=(3x^2+1)(3x^2-1)-x^2y-y$} is solved for *y*, the result is $y=ax+b$, for some constants *a* and *b*. Find *a* + *b*.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Round 3 Geometry: Solids and Volumes

1.)______{60, 15,135}____________

2.)_______{ 4860, 1440, 180 }____

3.) ____{48, 192, 108} _______

1.) A cone has horizontal base and its vertex lies vertically above the center of the base. The cone has height {8, 4, 12}, and its volume is {96π, 12π, 324π}. The lateral area of the cone (the surface area not including the base) is Aπ. What is A?

2.) A sphere of radius {9, 6, 3} cm is inscribed in a cube. The volume that is outside the sphere but inside the cube is $a - b\pi$ cm^3. What is a - b?

3.) A pyramid has a square base, and the base is horizontal. The height of the pyramid is {4, 8, 6}. A horizontal plane cuts the pyramid into two parts such that the volume of the top part is ½ of the volume of the bottom part. If the height of this plane above the base is *k*, then $k = a - b\sqrt[3]{c}$, where *a* and *b* are rational numbers and *c* is an integer that is not divisible by the cube of any prime number. Find the product of *a*, *b*, and *c*.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Round 4 Radical Expressions and Equations

1. ________ {8, 11, 13}___________

2. ________{448, 384, 576}

3. ________{71, 67, 63}_____________

1.)_ For how many integer values of K is $\{2 + \sqrt{K+3} - K, 4 + \sqrt{K+3} - K, 6 + \sqrt{K+3} - K\}$ a positive number?

2.) Suppose that $a_0, a_1, a_2, \ldots$ is a sequence of numbers such that $a_0 = x$, $a_1 = \sqrt{a_0}$, $a_2 = \sqrt{a_1}$, $a_3 = \sqrt{a_2}$, and so on. If $a_6 = \{2^7, 2^6, 2^9]$, then $x = 2^n$. Find n.

3.) For how many integers n with $0 \leq n \leq \{10000, 9000, 8000\}$ is $\sqrt{2n+1}$ rational?

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Round 5 Polynomials and Advanced Factoring

1. _______{8, 12 , 16 }____________

2. _______{75, 131, 35}___________

3. ________{7, 44, 159 }___________

1.) Let $f(x) = x^3 + Ax + B$ and suppose that $f(1) = \{3,5,7\}$ and $f(2) = \{15,20,25\}$. Find $|A| + |B|$.

2.

$$\{x^4 - 5x^3 + 17x^2 - 45x + K,$$
$$x^4 - 5x^3 + 24x^2 - 80x + K,$$
$$x^4 - 5x^3 + 12x^2 - 20x + K\}$$

factors to

$\{(x^2+9)(x^2+Bx+C), (x^2+16)(x^2+Bx+C), (x^2+4)(x^2+Bx+C)\}$.

Find K+B+C.

3.

A quartic polynomial $x^4 + Ax^3 + B\mathrm{x}^2 + Cx + D$, where *A*, *B*, *C*, *D* are integers, has {2+i and 1-2i , 3+i and 1-3i, 4+i and 1-4i} as two of its zeros, where $i = \sqrt{-1}$. Find A+B+C+D.

FAIRFIELD COUNTY MATH LEAGUE 2020-21

Match 6 Round 6 Counting and Probability

1.) _______{6,5,6}______

2.) ________{ 240, 80, 560}_________

3.) _____{31, 37, 9}__________

1.) ${}_{N}\mathrm{C}_{R}$ denotes the number of combinations of N objects taken R at a time. For how many of the {10,13,15} integer values of R, $0 \le R \le$ {9,12,14}, is $\{{}_{9}\mathrm{C}_{R}, {}_{12}\mathrm{C}_{R}, {}_{14}\mathrm{C}_{R}\}$ divisible by {9,12,14} ?

2.) The {12,10,14} members of a club consist of {6,5,7} married couples. A subset of 4 club members will be selected to represent the club at a conference. In how many ways can this be done if no person and his/her spouse may both be selected?

3) {Four nickels and six dimes, Three nickels and six dimes, Four nickels and five dimes} are placed in a bag, and five coins are drawn from the bag at random without replacement. The probability that the value of the coins is at least 40 cents is $\left\{\frac{A}{42}, \frac{A}{42}, \frac{A}{14}\right\}$. Find A.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 6 Team Round

1.) ________186_____ 4.) _______6_____________

2.) ________167_____ 5.) _____ 23_________

3.) _______16_______6.) _______31744____

1.) A, B, C, and D are the interior angles of a convex quadrilateral ABCD. The measure of the supplement of angle D is six degrees more than the measure of angle B. Find the sum of angles A and C.

2.) If $k = \sqrt[3]{3+\sqrt[3]{3+\sqrt[3]{3+\sqrt[3]{3+\ldots}}}}$ and k is real, what is $100k$ rounded to the nearest integer?

3.) A regular tetrahedron has volume $\frac{16\sqrt{2}}{3}$. The surface area is $K\sqrt{3}$ for some value of K. Find K.

4.) $x^3 + Ax^2 + Bx - 6$ factors into three binomials with integer coefficients. What is the absolute value of the sum of all possible values of A ?

5) When three standard six-sided dice with sides labeled 1 through 6 are rolled, the probability that the sum is 12 or 13 is $\frac{K}{108}$. Find K.

6) A softball player has a probability 0.4 of getting a hit in any at bat. She comes to bat 5 times in one game, and the results of her at-bats are independent. The probability that she gets at least 3 hits is K.
Find 100,000*K.