FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 5 Round 1 Arithmetic: Fractions & Exponents

1.)

2.)

3.)

1.) Let an "h-cube" be a cube whose volume is half a cubic centimeter. A number of h-cubes will now be piled up, one on top of the other. What is the smallest number of h-cubes that are needed in order to make a pile whose height is more than {16,13,12} centimeters?

2.) A fraction is written with n as its numerator and $n + \{6,10,15\}$ as its denominator. For how many integers n, with $1 \leq n \leq 100$, is the fraction reducible?

3.) If $\frac{(2^{4x})(3^{6y})}{(8^{2y})(9^{x})\left(\frac{1}{9^{-x}}\right)} = \left\{\frac{3}{2}, \frac{8}{27}, \frac{81}{16}\right\}$, find the value of $16x^2 - 48xy + 36y^2$.

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Match 5 Round 2
Algebra 1: Fraction Expressions and Equations

1.)

2.)

3.)

1.) If N liters of orange juice are shared equally between $\{6,8,9\}$ people, then each person gets $\{\frac{7}{8}, \frac{7}{12}, \frac{2}{3}\}$ of a liter more than if $N-2$ liters are shared equally between $\{8, 12, 15\}$ people. Find N.

2.) The equation $\frac{1}{n} - \frac{\{10,6,8\}}{n^2} + \frac{a}{n^3} = 0$, where a is a nonzero constant, has only one solution for n. Find the value of a.

3.) Let k be that constant such that one of the solutions of the equation $\frac{2x+3}{x+k} = \frac{x-1}{3x+4}$ is $x = \{2.4,1.5,1.2\}$. Find the other solution for x.

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Match 5 Round 3
Geometry: Circles

1.)

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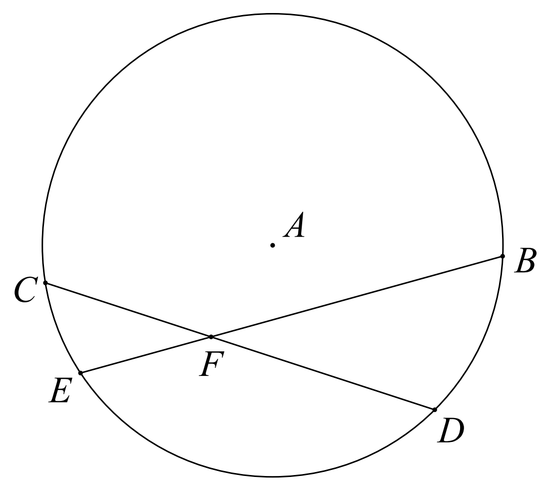
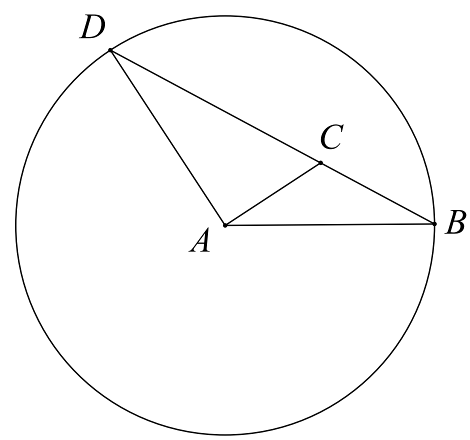
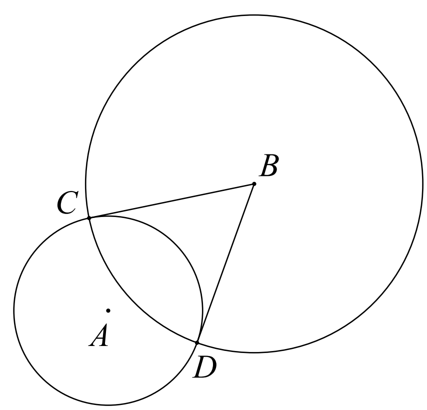
1.) See the diagram. The chords $\overline{CD}$ and $\overline{BE}$ intersect at point F. If $m\angle CFB = 144°$ and the measure of arc EC is $\{20,24,28\}$ degrees, find the degree measure of arc BD.



2.) In the diagram, A is the center of the circle, $\angle DAB \cong \angle ACB$, $BC = \{9,7,5\}$, $BD = \{22,18,14\}$, and the area of the circle is $m\pi$. Find the value of m.



3.) See the diagram. The circle with center B intersects the circle with center A at points C and D such that $\overline{BC}$ and $\overline{BD}$ are tangent to circle A. If the area of circle A is $\{30\pi, 50\pi, 40\pi\}$, $CD = \{6\sqrt{2}, 4\sqrt{10}, 4\sqrt{6}\}$ and the area of circle B is $k\pi$, find the value of k.



FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 5 Round 4 Algebra 2: Quadratic Equations and Complex Numbers

1.)

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1.) Let $f(z) = z^2 - \{4,2,6\}z$. There exists a complex number w such that $f(w) = f(i)$. Let $w = a + bi$ where a and b are nonzero real numbers. Find the value of a.

2.) Let k be the complex constant such that the discriminant of the equation $z^2 - 3iz + k = 0$ is $\{6 + i, 2 + 5i, 4 - i\}$. If $k = a + bi$, where a and b are real, find $|a + b|$.

3.) The equation $z^2 + az + b = 0$, where a and b are complex constants, has two zeros, z_1 and z_2, Both zeros are natural number powers of $\{1 + 2i, 2 + i, 1 + 3i\}$. If $|z_1| = \{5\sqrt{5}, 5\sqrt{5}, 10\}$ and $|z_2| = \{25, 25, 10\sqrt{10}\}$, find $|a|^2$.

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Match 5 Round 5 Trig and Advanced Topics: Trigonometric Equations

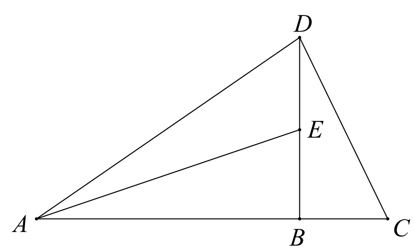
1.)

2.)

3.)

1.) How many solutions x are there to the equation $\cos\left(\{10,8,9\}x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, with $0 \leq x \leq 2\pi$?

2.) In the diagram, $\overline{BD}$ is perpendicular to $\overline{AC}$. $AD = \{4,5,6\}$ and $DC = 2$. Let $m\angle DAE = m\angle EAB = m\angle BDC = \theta$, where $0 < \theta < 45°$. Find θ, rounding your answer to the nearest degree. (Do not include a unit.)



3.) Given that

$$\{(k+3)\sec^2 x - 2k = 3\tan^2 x + 3\sec x \tan x,$$
$$(k+1)\sec^2 x - (2k-4) = 3\tan^2 x + 3\sec x \tan x,$$
$$(k+4)\sec^2 x - (2k+2) = 3\tan^2 x + 3\sec x \tan x\},$$

where k is a constant, the product of all possible values of $\sin x$ is $-\frac{4}{11}$. Find the value of k.

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Match 5 Round 6
Miscellaneous: Sequences and Series

1.)

2.)

3.)

1.) The first three terms of an arithmetic sequence are $\{2.2, 2.8, 3.4, \ldots;\ 2.6, 3.4, 4.2, \ldots;\ 3.8, 4.2, 4.6, \ldots\}$. How many terms of the sequence are less than 2021 and are integers?

2.) Let $a_0, a_1, a_2, \ldots$ be an arithmetic sequence with common difference d. Let $b_0, b_1, b_2, \ldots$ be a geometric sequence with common ratio $\{1.2, 1.6, 1.8\}$. If $a_0 = b_0$ and $a_2 = b_2$, what is $100\frac{d}{a_0}$?

3.) An infinite geometric series has the property that the first term minus the second term is $\{225{,}196{,}245\}$ and the first term minus the third term is $\{75{,}49{,}147\}$. What is the sum of the series?

Team Round

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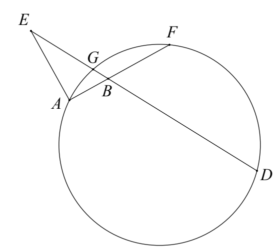
1.)
2.)
3.)
4.)
5.)
6.)

1.) Let $x_1, x_2, x_3 \ldots$ be a sequence of numbers such that $x_1 = 3$ and, for $n \geq 1$, $x_{n+1} = \frac{1}{1+x_n}$. Find the reciprocal of the product $x_1 * x_2 * \ldots * x_8$.

2.) Consider the equation $x + \frac{1}{a+\frac{1}{x}} = b$, where a and b are constants. If the solutions for x have the same absolute value and $a + b = 5$, find the value of $a^2 + b^2$.

3.) See the diagram (not necessarily drawn to scale). Chords $\overline{DG}$ and $\overline{AF}$ intersect at point B and $\overline{AE}$ is perpendicular to $\overline{AF}$. $BG = 2$, $BD = 10$, $AE = BF$, and $GE = 7$. Find AF.



4.) If a and b are positive real numbers such that $\left|\frac{5+ai}{b+3i}\right| = 1$ and $|a + bi| = \sqrt{65}$, find $2ab$.

5.) If $\theta = \tan^{-1}(\frac{1}{5})$ is a solution to the equation $\tan(\theta + A) = \frac{4}{7}$, find $27\sec^2(A)$.

6.) Consider a geometric series and an arithmetic sequence, where the first term of the geometric series is equal to the first term of the arithmetic sequence, and the 50th term of the geometric series is equal to the 50th term of the arithmetic sequence. The common difference of the arithmetic sequence is -10. If the geometric series converges to a value that is $\frac{9}{7}$ times its first term, find the sum of the first 49 terms of the geometric series.